

# Stability of Flat Space to Singular Instantons

Neil Turok\*

*DAMTP, Silver St, Cambridge, CB3 9EW, U.K.*

(February 7, 2008)

## Abstract

Hawking and the author have proposed a class of singular, finite action instantons for defining the initial conditions for inflation. Vilenkin has argued they are unacceptable. He exhibited an analogous class of asymptotically flat instantons which on the face of it lead to an instability of Minkowski space. However, all these instantons must be defined by introducing a constraint into the path integral, which is then integrated over. I show that with a careful definition these instantons do not possess a negative mode. Infinite flat space is therefore stable against decay via singular instantons.

## I. INTRODUCTION

Hawking and the author recently discovered a new class of instantons which could be relevant for inflationary cosmology. For a generic inflationary potential there exists a one parameter family of finite action solutions to the Euclidean field equations, which are the natural deformation of the four sphere solution to pure gravity with a cosmological constant. They may be analytically continued to give an open inflating universe. These instantons possess a singular boundary of zero size, but it was argued in [1,2] that quantum fluctuations are well defined in its presence. The expectation was expressed that low energy phenomena would still be uniquely determined in the presence of the singularity. This has subsequently been confirmed by calculations of the perturbation spectra [3,4].

Vilenkin has criticised of the use of such instantons [5]. He constructed a related class of asymptotically flat singular instantons which on the face of it lead to the nucleation of holes in flat space. He estimated the rate per unit spacetime volume and concluded it could be arbitrarily large. As a result, he advocated excluding such instantons just because they are singular. I don't think this is a tenable conclusion. Typical field configurations contributing to the path integral are non-differentiable Brownian random walks, so to ignore singular configurations is to ignore essentially everything. If the Euclidean action for quantum gravity cannot by itself suppress such horrors as the decay of flat space, it is unlikely that the gravitational path integral can be made sense of at all. However, a clue to the correct interpretation of these instantons was an observation due to Wu [9] that even though they are solutions of the field equations (away from the singularity), they are not stationary points

---

\*email:N.G.Turok@damtp.cam.ac.uk

of the action. Their action depends linearly on a parameter which governs the strength of the singularity. Therefore they must be treated as constrained instantons [2].

In this Letter I show that a careful treatment of constrained singular instantons removes Vilenkin's instability. The simplest argument that flat space is stable employs conservation of energy. The positive energy theorem, states that any *regular* asymptotically flat space which has zero ADM energy is flat [15]. If energy is conserved, flat space cannot decay. However, Vilenkin's instantons are singular and have zero ADM energy [7], so this argument cannot unfortunately be applied.

We must therefore compute the the amplitude for flat three space to propagate into itself. If this possesses an imaginary part we can conclude that flat space is unstable. As usual we Wick rotate the path integral to Euclidean time, and sum over asymptotically flat Euclidean four geometries. The only regular solution of the Einstein equations with these boundary conditions is flat space. That is the only genuine instanton and it possesses no negative mode [11].

What about fluctuations about constrained instantons? Think of the four geometry space as a fluctuating rubber sheet. If we push on a sheet with a ring of wire, we can deform it. The region of the sheet outside the ring is a nontrivial solution of the equations of motion, and the region inside it will be flat. One can take the radius of the ring to zero, leaving a 'spike' on the surface of the sheet. I claim that this is the correct interpretation of Vilenkin's instantons. Note that even though there is nothing to 'push' on spacetime, configurations like this are inevitably produced as quantum fluctuations. We should then check for stability around these 'spiky' configurations.

It is easy to see that such configurations inevitably contribute to the path integral. I am not saying that they are a significant contribution, nor that they lead to any sensible approximation scheme for it. But they are *there*. The point is that one can simply introduce the identity  $1 = \int dC \delta(\mathcal{C} - C)$  into the functional integral, for some appropriate operator  $\mathcal{C}$ . The simplest example is for massive scalar field theory, for which  $\mathcal{C}$  could just be the value of the field at a given point. In this case as in ours, for each  $C$  there is a nontrivial classical solution. For a massive scalar field the solution would just be of the Yukawa potential form. The idea is to perform the functional integral over fields first, in a saddle point approximation, before finally integrating over  $C$ . This procedure is well established in quantum mechanics and in the theory of instantons in gauge theories with spontaneous symmetry breaking [6]. In our case, we want  $\mathcal{C}$  to be a local property of the geometry, which roughly speaking measures how 'spiky' it is. We define  $\mathcal{C}$  on a surface  $\Sigma_c$  of radius  $a$  about a given point. For fixed  $a$  I shall show that such an instanton exists. Outside  $\Sigma_c$  it looks like one of Vilenkin's instantons. Inside it is just flat space. As  $a$  tends to zero, nothing in the definition becomes singular and the Euclidean action remains finite <sup>1</sup>.

Is flat space is unstable against such instantons? I think this is a nontrivial question. If the answer were positive, one would I think be forced to discard the the path integral for

---

<sup>1</sup> Strictly speaking, there is of course no need to take the limit as  $a$  goes to zero. However, the description of the instantons simplifies in that limit. This is particularly important in the cosmological case [1] where one is interested in the Lorentzian continuation of the instanton. Only in the limit as  $a$  goes to zero will the continued spacetime be real.

quantum gravity as a sick theory. Luckily, as I show, that turns out not to be the case.

How do we check for an instability? The signature is the imaginary contribution to the ‘self-energy’ diagram for flat space, mentioned above. The question is when we integrate over these instantons and fluctuations about them, whether any factors of  $i$  emerge. If for fixed  $C$  there is a negative mode in the field fluctuations, we would be faced with an integral of the form  $\int dx e^{+x^2}$ , which could only be defined by rotating the contour. This would introduce an  $i$ . The only other way we could get an  $i$  is if the integral over  $C$  performed at the end of the calculation were not convergent, and required a similar rotation to define it. For example, if we just inserted  $1 = \int dC \delta(C - x)$  into the above integral over  $x$ , the  $x$  integral would be no problem, but the  $i$  would reappear in the integral over  $C$ .

For Vilenkin’s instantons the second possibility is easy to exclude. There are no instantons satisfying the boundary conditions at infinity for negative  $C$ . But for positive  $C$ , the Euclidean action increases linearly with  $C$ . So the integral over  $C$  involves  $\int_0^\infty dC e^{-C}$ , which is perfectly convergent and does not yield a factor of  $i$ . Physically, what this says is that the Euclidean gravitational action, like the energy of a rubber sheet, suppresses geometries with spikes in them.

The analysis for fluctuations about such instantons is quite technical. As is well known, the Euclidean Einstein action is not positive semidefinite, so the path integral must be defined with care. However, for perturbations around instantons in pure gravity the procedure is well understood. One has to use a method which clearly separates out the conformal mode [10], [11], [12]. I shall concentrate here on the simplest singular instanton described by Vilenkin, involving a massless scalar field coupled to gravity. Such instantons may be reinterpreted *à la* Kaluza-Klein as the Euclidean Schwarzschild solutions of five dimensional gravity [13]. As well as being analytically known, the five metric is regular, which allows a clearcut analysis of fluctuations. Furthermore, because this is just pure gravity, we can use well established gauge-fixing procedures ([11] and references therein).

I want to emphasise that I am *not* concerned here with the five dimensional theory or its geometry. I am studying four dimensional constrained instantons, and *only* using the five dimensional variables as a trick for gauge fixing in the path integral. Nevertheless, from what is already known about five dimensional gravity, there is a clear candidate for the negative mode and it only involves the variables of the four dimensional theory [14]. I shall construct it explicitly in order to prove that it does not exist in the situation of interest.

Witten argued that the five dimensional Euclidean Schwarzschild solution had a negative mode in analogy with the four dimensional case. The four dimensional version is thought to describe the nucleation of black holes in hot flat space. In five dimensions Witten argued that it describes the decay of the Kaluza Klein vacuum. I shall not be concerned with any of these interpretations here, nor with the five dimensional geometry. I merely use the five dimensional variables as a convenient choice for performing the four dimensional functional integral. The constrained variable  $\mathcal{C}$  has a simple expression in terms of five dimensional variables, and is not permitted to fluctuate in the path integral over fields. This condition turns out to eliminate the candidate negative mode mentioned above. Without the negative mode, the instantons do not yield an imaginary contribution to flat space to flat space amplitude. Flat space is therefore stable against decay via singular instantons.

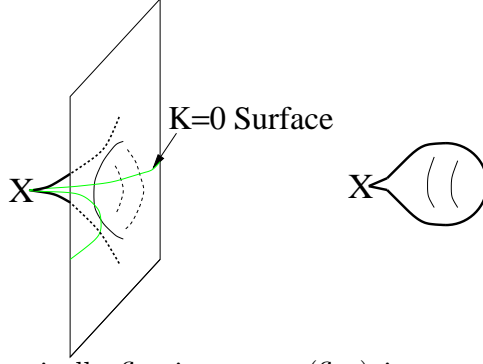


FIG. 1. Vilenkin's asymptotically flat instanton (flat) is compared with the Hawking-Turok instanton (right). Euclidean time runs upwards on Vilenkin's instanton. If one identifies the lower edge of the diagram (infinite negative Euclidean time) with flat three dimensional spacetime and cuts the diagram horizontally through the singularity  $X$  on the three surface shown up on which  $K_{ij} = 0$ , and continue Euclidean time to obtain a Lorentzian manifold with a hole in it. Vilenkin claimed that if singular instantons were permitted in the path integral, flat space would be unstable towards developing holes in this manner.

## II. VILENKIN'S INSTANTONS

Vilenkin's instantons are solutions to the field equations for four dimensional gravity coupled to a massless scalar field. They possess a singularity, and a conformal transformation reveals it to be a boundary in the form of a three sphere of zero size [1]. The boundary is perhaps more disturbing than the presence of a singularity, and this might lead one to simply exclude such configurations from the path integral by fiat [7]. However, I shall now show that Vilenkin's instantons exist as a well defined limit of a regular class of constrained instantons, with no boundary. So defined, they inevitably contribute to the path integral.

The action under consideration is usually written

$$S_E = \int d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} + \frac{1}{2}(\partial\phi)^2 \right] - \int_{\Sigma} d^3x \sqrt{h} \frac{K}{8\pi G}. \quad (1)$$

where last 'surface' term is introduced to remove second derivatives from the gravitational term. The induced three metric on  $\Sigma$  is  $h_{ij}$  and  $K = K_{ij}h^{ij}$  is the trace of the second fundamental form. In coordinates where  $\Sigma$  is a surface of constant  $\tau$ , we have  $K_{ij} = N^{-1}(-\partial_{\tau}h_{ij} + N_{(i;j)})$  with  $N$  and  $N_i$  the lapse and shift functions. The surface term can be thought of as the rate of change of the proper volume of constant  $\tau$  surfaces with respect to proper time. That is, it is the Euclidean version of the Hubble constant times the three volume.

We are interested in four manifolds with no boundary other than that at infinity. The instantons of interest will be solutions of the field equations everywhere except on a particular three surface  $\Sigma_c$ , where constraint is imposed. The action is a local integral of terms involving at most first derivatives, and for the solutions of interest the action density will be spread over space. However, if we integrate by parts to write the action as above, the bulk term actually vanishes since it is proportional to the trace of the Einstein equations. But the constrained solutions will have discontinuous first derivatives normal to  $\Sigma_c$ , and this leads

to a contribution from the difference in normal derivatives of the metric across  $\Sigma_c$ , as well as a boundary term from the asymptotically flat surface at infinity.

The constraint is defined as follows. We would like a variable which measures the strength of ‘spikes’ on the manifold. The natural one to choose is just the surface term in the Einstein action. Around a coordinate point on the manifold, draw a three surface  $\Sigma_c$  of geodesic radius  $a$ . The constrained variable is then

$$\mathcal{C} = \left[ \int_{\Sigma_c} d^3x \sqrt{h} \frac{K}{8\pi G} \right]_+^-, \quad (2)$$

where contributions from both sides of  $\Sigma_c$  are included. Each interpolating four geometry in the path integral will have some value for  $\mathcal{C}$ , and the functional delta function we introduce splits the path integral up accordingly. Only geometries with discontinuous first derivatives possess nonzero  $\mathcal{C}$ . However this includes nearly all geometries since the class of metrics with continuous first derivatives is a set of measure zero.

Within each class of metrics so defined, there is a corresponding  $O(4)$  invariant instanton. If we write the metric in general  $O(4)$  invariant form

$$ds^2 = n^2 d\sigma^2 + b^2(\sigma) d\Omega_3^2 \quad (3)$$

where  $d\Omega_3^2$  is the metric for  $S^3$ , then the action (1) reduces to

$$S_E = \int d\Omega_3 \int d\sigma \left( -\frac{3}{8\pi G} (n^{-1} b b_\sigma^2 + n b) + \frac{1}{2} n^{-1} b^3 \phi_\sigma^2 \right). \quad (4)$$

The classical field equations are

$$(n^{-1} b^3 \phi_\sigma)_\sigma = 0 \quad b_\sigma^2 = \frac{4\pi G}{3} b^2 \phi_\sigma^2 + n^2. \quad (5)$$

where subscripts denote derivatives. Our instantons will be solutions to the classical field almost everywhere. The action is then given by

$$S_E = -\frac{2\pi^2}{8\pi G} n^{-1} (b^3)_\sigma |^\infty + \left[ \frac{2\pi^2}{8\pi G} n^{-1} (b^3)_\sigma \right]_-^+ \quad (6)$$

where the second term is just the constrained variable (2).

Now we discuss solutions to the field equations. We redefine  $\sigma$  so that  $n = 1$ . The scalar field equation possesses the general solution  $\phi_\sigma = A/b^3(\sigma)$ , with  $A$  an arbitrary constant. Asymptotic flatness requires that  $b \sim \sigma$  at large  $\sigma$ . Thus  $b$  satisfies

$$b_\sigma^2 - \frac{4\pi G A^2}{3} b^{-4} = 1, \quad (7)$$

the equation for a particle of unit energy in a negative  $b^{-4}$  potential. The exterior solution is unique up to a constant shift in  $\sigma$ . Take our constraint surface to be at  $\sigma_c$ . The interior solution is flat space, with  $b(\sigma) = (\sigma - \sigma_0)$  where  $\sigma_0$  is the location of the origin in spherical coordinates, determined by matching  $b$  at  $\sigma_c$ . By shifting  $\sigma$  we can set  $\sigma_0 = 0$ . The scalar field is constant in the interior region.

With our instantons defined for finite  $a$ , we take the limit as  $a$  tends to zero. The exterior solution is that for the ‘singular instanton’ solutions of [1], [5]. The interior is flat space in Vilenkin’s case, and nearly so in the cosmological case. There is a contribution to the Einstein action from the difference (6) evaluated across  $\sigma_c$ . From (7) in the limit of small  $b$  this contribution to the action is  $\sqrt{\frac{3}{2}}|A|2\pi^2/\sqrt{8\pi G}$ , which is strictly positive, and increasing with  $|A|$ . Additionally, there is a negative infinite contribution to the action from the surface at infinity. However in the path integral one must normalise the one instanton contribution relative to the no-instanton contribution. This means one has to subtract the surface term appropriate to flat space, which is  $-(2\pi^2/8\pi G)\partial_R R^3 = -(2\pi^2/8\pi G)3b^2$ , since at large distances we identify  $b$  with the radius  $R$ . The equation (7) has solution  $b \sim \sigma + o(\sigma^{-3})$  at large  $\sigma$ , so after subtraction the surface contribution from infinity is actually zero.

To summarise: Vilenkin’s instantons may be defined as a limit of constrained instantons which are nonsingular (albeit with discontinuous first derivatives) and have no boundary. As such, they are legitimate contributions to the path integral which must be present in quantum gravity (and therefore presumably in the real world). But far from signalling an instability, the Euclidean action monotonically increases as we increase the strength of the ‘spike’. As claimed in the introduction, the fact that the action increases monotonically with  $C$  guarantees that the integration over  $C$  is convergent, and no factors of  $i$  emerge. This is just as for the surface of a balloon poked with a pencil: the energy increases. If we take the pencil away the surface returns to being smooth. In our case there is no pencil, but quantum mechanical vacuum fluctuations continually produce configurations close to Vilenkin’s instantons. They come and go, but never cause permanent damage.

### III. FLUCTUATIONS ABOUT CONSTRAINED INSTANTONS

As is well known, four dimensional gravity with a massless scalar field can be obtained via dimensional reduction of five dimensional gravity *à la* Kaluza Klein. The five dimensional metric is given in terms of the four dimensional metric and the scalar field as

$$g_{\mu\nu}^{(5)} = e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}} g_{\mu\nu}^{(4)}, \quad g_{55}^{(5)} = e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{Pl}}}, \quad g_{\mu 5}^{(5)} = 0, \quad (8)$$

where  $g_{\mu\nu}^{(4)}$  is the four metric in the Einstein frame,  $\mu, \nu = 0, 1, 2, 3$  and  $M_{Pl}^2 = (8\pi G)^{-1}$ . The five dimensional field equations  $R_{ab}^{(5)} = 0$ ,  $a, b = 0..5$  reduce to those for a massless field  $\phi$  coupled to four dimensional gravity, and the  $O(4)$  invariant solution described above is in fact just the five dimensional Euclidean Schwarzschild solution. Unlike its Lorentzian counterpart, the latter is perfectly regular. It is intriguing that the singularity of the four dimensional metric  $g_{\mu\nu}^{(4)}$  disappears when we change to the five dimensional one, but for present purposes this is a mere calculational convenience.

We are interested in  $O(4)$  symmetric metrics, of the form

$$ds^2 = N^2(\tau)d\tau^2 + R^2(\tau)d\Omega_3^2 + r^2(\tau)d\phi^2. \quad (9)$$

For the Euclidean Schwarzschild solution  $N$ ,  $R$  and  $r$  are given by

$$N_0 = 1 \quad R_0^2 = C + \tau^2 \quad r_0^2 = C\tau^2/(C + \tau^2). \quad (10)$$

but when we consider fluctuations we shall perturb  $N$ ,  $R$  and  $r$ .  $C$  is an integration constant, related to the ‘mass’ of the Schwarzschild solution ( $R_0(\tau)$  is the usual Schwarzschild radial variable), or in the Kaluza Klein interpretation to the radius of the periodic dimension at infinite  $\tau$ .

Comparing equations (3), (8,) and (9), we can construct the 5d to 4d dictionary,

$$r = e^{-\sqrt{\frac{2}{3}}\phi/M_{Pl}} \quad R = be^{+\frac{1}{2}\sqrt{\frac{2}{3}}\phi/M_{Pl}} \quad Nr^{\frac{1}{2}}d\tau = nd\sigma \quad C^3 = 16\pi G|A|^2/3 \quad (11)$$

The boundary term we are interested in is the four dimensional one, given in equation (6). When expressed in five dimensional variables, this is

$$\int d\Omega_3 \frac{1}{8\pi G_4} \left[ N^{-1} r^{-\frac{1}{2}} \partial_\tau (R^3 r^{\frac{3}{2}}) \right]_{\tau=0}. \quad (12)$$

Note that this is *not* the boundary term for five dimensional gravity - the latter would have the  $r^{\frac{3}{2}}$  replaced by  $r$  and  $r^{-\frac{1}{2}}$  replaced by unity (cf. ref. [13]).

#### IV. NO NEGATIVE MODE

We now consider perturbations of the background solution  $g_{ab}^B$  discussed above. We set  $g_{ab} = g_{ab}^B + h_{ab}$ , and compute the Euclidean Einstein action to second order in  $h_{ab}$ . In addition to the problem of gauge fixing, the task of finding negative modes is complicated by the fact that the gravitational action for conformal deformations of the metric is unbounded below. At first sight there appear to be an infinite number of negative modes. However, these are unphysical. In the context of pure gravity and for perturbations around classical instantons the resolution of the problem has been well understood for some time. One must be careful to separate the fluctuations of the conformal factor from the transverse traceless fluctuations in the path integral [11] [12].

After a suitable gauge fixing term is added, the path integral over the conformal factor decouples from that over the transverse traceless metric perturbations. The latter are described by a quadratic action involving the Lichnerowicz operator, which involves the Riemann tensor of the background solution. The problem then is to find the eigenmodes of the Lichnerowicz operator,

$$-\square h_{ab} - 2R_{acbd}g_B^{ce}g_B^{df}h_{ef} = \lambda h_{ab} \quad (13)$$

where the perturbation  $h_{ab}$  is transverse and traceless, so  $\Delta^a h_{ab} = g_B^{ab} h_{ab} = 0$ . Here  $\Delta^a$  and  $\square$  are the usual covariant derivatives and Laplacian constructed from the background metric. If equation (13) has a normalisable solution for negative  $\lambda$ , the instanton has a genuine physical negative mode.

I shall consider only S-wave perturbations of the metric, since higher angular momentum perturbations are guaranteed to have greater  $\lambda$ . The most general S-wave perturbation may be written in terms of the variables  $N$ ,  $R$ , and  $r$  defined in (9) and the background solution (10) as

$$N^2 = 1 + 2f \quad R^2 = R_0^2(1 + 2g) \quad r^2 = r_0^2(1 + 2h) \quad (14)$$

where tracelessness and transversality read

$$f + 3g + h = 0 \quad \dot{f} + \gamma^{-1}\dot{\gamma}f - 3(\dot{R}_0/R_0)g - (\dot{r}_0/r_0)h = 0. \quad (15)$$

and (13) reads

$$-\ddot{f} - \gamma^{-1}\dot{\gamma}f + 6\left[\frac{\ddot{R}_0}{R_0} - \left(\frac{\dot{R}_0}{R_0}\right)^2\right]g + 2\left[\frac{\ddot{r}_0}{r_0} - \left(\frac{\dot{r}_0}{r_0}\right)^2\right]h + \left[6\left(\frac{\dot{R}_0}{R_0}\right)^2 + 2\left(\frac{\dot{r}_0}{r_0}\right)^2\right]f = \lambda f \quad (16)$$

where  $\gamma = R_0^3 r_0$ . Equations (15) can be used to eliminate  $g$  and  $h$  from (16). Using  $R_0(\tau)$  and  $r_0(\tau)$  from (10) we find

$$-\ddot{f} - \frac{3 + 10\tau^2 - 5\tau^4}{\tau(1 - \tau^4)}\dot{f} - \frac{20}{(1 - \tau^4)}f = \lambda f. \quad (17)$$

This equation possesses regular singular points at  $\tau = 0, 1$  and  $\infty$ . Regularity at  $\tau = 1$  requires that  $\dot{f}(1) = -\frac{5}{2}f(1)$ . One can use a shooting technique to search for the lowest eigenvalue  $\lambda$ . One starts from  $\tau = 1$  with  $f = 1$  and  $\dot{f} = -\frac{5}{2}$ . Given  $\lambda$ , the solution is propagated to large  $\tau$  by solving the differential equation. One adjusts  $\lambda$  so that the solution goes to zero at infinite  $\tau$ . Having found  $\lambda$ , the solution for  $\tau < 1$  is found by solving the equation with boundary condition  $\dot{f}(0) = 0$ . The damping term in the equation is singular and ensures that as  $\tau$  approaches unity the correct relation between  $\dot{f}$  and  $f$  is satisfied. Finally, rescaling the  $0 < \tau < 1$  part of the solution to match the  $\tau > 1$  part at  $\tau = 1$ , one has the complete eigenfunction. With this procedure I found only one negative value for  $\lambda$ , namely  $\lambda = -1.25$ , with small error. The negative mode is shown in Figure 6.

Now we have discovered the single allowed negative mode, the key question is whether it is allowed by the constraint on (12). When we perturb the instanton, only perturbations leaving (12) unperturbed are allowed. This condition reads

$$\delta[N^{-1}r^{-\frac{1}{2}}\partial_\tau(R^3r^{\frac{3}{2}})]_{\tau=0} \propto [3g + h - f]_{\tau=0} = 0, \quad (18)$$

where I used  $r_0 \sim \tau$  and  $R_0 \sim \text{const}$  as  $\tau$  goes to zero. However the traceless condition imposes  $f + 3g + h = 0$  and transversality imposes  $f = h$  at  $\tau = 0$  (see (15)). These conditions together require  $f = 0$  at  $\tau = 0$ . The negative mode we have found, which is the only one, is therefore excluded. Note that the constraint on (12) has no effect on any of the higher  $S^3$ -dependent modes, since these give no contribution to the boundary term when integrated over  $\Sigma$ .

I conclude that if the instantons under consideration are properly treated as constrained instantons, as they must be, they do not possess a negative mode and therefore do not lead to the decay of flat spacetime. I believe the calculations reported above should be interpreted as a nontrivial test of the quantum gravitational path integral, and as such they are a good sign. Furthermore, the above discussion of constrained instantons does, I think, considerably clarify their interpretation in the cosmological context [17].

### Acknowledgements

I am indebted to M. Bucher, S. Gratton, H. Reall, S.W. Hawking, M. Perry and T. Wiseman for helpful discussions of this problem.



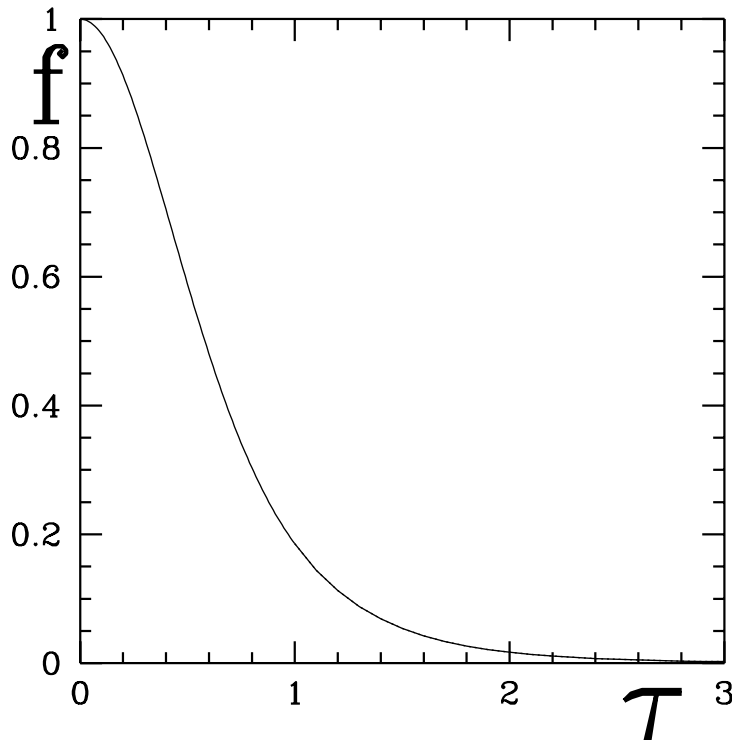


FIG. 2. The negative mode for the five dimensional Euclidean Schwarzschild solution. This mode is eliminated from the four dimensional theory by the constraint required for the existence of the instanton.

## REFERENCES

- [1] S.W. Hawking and N. Turok, Phys. Lett. **B425** (1998), 25, hep-th/9802030.
- [2] N. Turok and S.W. Hawking, Phys. Lett. **B432**, (1998) 271, hep-th/9803156.
- [3] S. Gratton and N. Turok, DAMTP preprint, in preparation (1999).
- [4] T. Hertog and N. Turok, DAMTP preprint, in preparation (1999).
- [5] A. Vilenkin, Phys.Rev. D57 (1998) 7069, hep-th/9803084.
- [6] I. Affleck, Nuc. Phys. **B191** (1981) 429.
- [7] I thank H. Reall for these observations.
- [8] R. Schoen and S.T. Tau, Phys. Rev. Lett. **42** (1979) 547.
- [9] Z.C. Wu, Beijing preprint, hep-th/9803121.
- [10] S.W. Hawking, in *General Relativity, an Einstein Centenary Survey*, Ed. S.W. Hawking and W. Israel, Cambridge University Press (1979).
- [11] D.J. Gross, M.J. Perry and L.G. Yaffe, Phys. Rev. D25, 330 (1982).
- [12] E. Mottola, J. Math. Phys. **36** (1995) 2470, hep-th/9502109.
- [13] J. Garriga, Barcelona preprint, hep-th/9803210.
- [14] I thank S.W. Hawking for emphasising the importance of studying this question.
- [15] R. Schoen and S.T. Yau, Phys. Rev. Lett. **42**, 547 (1979).
- [16] E. Witten, Nuc. Phys. **B195** (1982) 481.
- [17] N. Turok, DAMTP preprint, in preparation (1999).